TA Session 4
Dynamic Discrete Choice: CCP

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Using CCP to Represent Differences in Continuation Values

$$
\begin{aligned}
v_j(x_t) &= u_j(x_t) + \beta EV(x_t) \\
\Rightarrow v_0(x_t) - v_1(x_t) &= u_0(x_t) - u_1(0) + \beta EV(x_t) - \beta EV(0) \\
&= u_0(x_t) - u_1(0) \\
&\quad + \beta \ln \left[e^{v_0(x_{t+1})} + e^{v_1(x_{t+1})} \right] - \beta \ln \left[e^{v_0(1)} + e^{v_1(1)} \right] \\
&= u_0(x_t) - u_1(0) + \beta \ln \frac{e^{v_0(x_{t+1})} + e^{v_1(x_{t+1})}}{e^{v_0(1)} + e^{v_1(1)}} \\
&\overset{(*)}{=} u_0(x_t) - u_1(0) + \beta \ln \frac{e^{v_0(x_{t+1}) - v_1(x_{t+1})} + 1}{e^{v_0(1) - v_1(1)} + 1} \\
&= u_0(x_t) - u_1(0) + \beta \ln \frac{p_1(1)}{p_1(x_{t+1})}
\end{aligned}
$$

where $(*)$ is implied by the renewal property $v_1(x_{t+1}) = v_1(1)$, and $d_t = 1 \Rightarrow x_t = 0, x_{t+1} = (0, 1, 2) \cdot (\varphi_0, \varphi_1, \varphi_2)'$.

Invertible Mapping between CCP and Differences in Continuation Values

- Therefore,

$$p_1(x) = \frac{1}{1 + e^{v_0(x) - v_1(x)}} = \frac{1}{1 + e^{u_0(x_t) - u_1(0) + \beta \ln \frac{p_1(1)}{p_1(x_{t+1})}}}$$

- Intuitively, the CCP for the current replacement is the CCP for a static model with an offset term.
- The offset term accounts for differences in continuation values using future CCPs that characterize optimal future replacements.
- An important implication of the invertible mapping is that estimation can be done without computing the value functions.

CCP Estimation

- CCP estimator in two stages:
  1. Estimate non-parametrically for $p_1(x)$ and the transitions
     - Estimate $p_1(x)$ from the relative frequencies:

$$\hat{p}_1(x) \equiv \frac{\sum_i \sum_t \mathbb{1}\{d_{it} = 1\} \cdot \mathbb{1}\{x_{it} = x\}}{\sum_i \sum_t \cdot \mathbb{1}\{x_{it} = x\}}$$

  2. Solve for utility parameters θ
     - Substitute $\hat{p}_1(x)$ into the likelihood as incidental parameters to estimate θ[1] with a logit:

$$\frac{d_1 + d_0 \cdot e^{u_0(x_t;\theta_0) - u_1(0;\theta_1) + \beta \ln \frac{\hat{p}_1(1)}{\hat{p}_1(x_{t+1})}}}{1 + e^{u_0(x_t;\theta_0) - u_1(0;\theta_1) + \beta \ln \frac{\hat{p}_1(1)}{\hat{p}_1(x_{t+1})}}}$$

       where $x_{t+1} = (0, 1, 2) \cdot (\varphi_0, \varphi_1, \varphi_2)'$ given $d_t = 1$.
     - Correct the std. err. for θ induced by the first stage estimates of $\hat{p}_1(x)$.

[1] The term $\ln \frac{\hat{p}_1(1)}{\hat{p}_1(x_{t+1})}$ enters the logit as an individual specific component of the data, thus β enters the logit in the same way as the utility parameters θ. You can also estimate β in this estimation.

CCP Estimation vs. Full Solution Approach

- The full solution approach
  - The full solution approach computes the optimal decision rule, it's computationally intensive especially if you have a rich state space.
  - While it may be theoretically possible to parametrically identify dynamic models with data sets that only track the first few periods of the decision maker's problem.
- The CCP approach
  - Instead of computing continuation values by solving the dynamic problem for all elements in the state space, the CCP approach "measures" continuation values using a function of CCPs.
  - The CCP estimator is faster but less versatile. It requires samples to be drawn from the population of all possible histories.

References

- Hotz, V. J., & Miller, R. A. (1993). Conditional choice probabilities and the estimation of dynamic models. The Review of Economic Studies, 60(3), 497-529.